
**Information technology — Security
techniques — Cryptographic techniques
based on elliptic curves —**

**Part 5:
Elliptic curve generation**

*Technologies de l'information — Techniques de sécurité — Techniques
cryptographiques fondées sur les courbes elliptiques —*

Partie 5: Génération de courbes elliptiques

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Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work. In the field of information technology, ISO and IEC have established a joint technical committee, ISO/IEC JTC 1.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of the joint technical committee is to prepare International Standards. Draft International Standards adopted by the joint technical committee are circulated to national bodies for voting. Publication as an International Standard requires approval by at least 75 % of the national bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO and IEC shall not be held responsible for identifying any or all such patent rights.

ISO/IEC 15946-5 was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology*, Subcommittee SC 27, *IT Security techniques*.

ISO/IEC 15946 consists of the following parts, under the general title *Information technology — Security techniques — Cryptographic techniques based on elliptic curves*:

- *Part 1: General*
- *Part 5: Elliptic curve generation*

Introduction

Some of the most interesting alternatives to the RSA and $F(p)$ based systems are cryptosystems based on elliptic curves defined over finite fields. The concept of an elliptic curve based public-key cryptosystem is rather simple:

- Every elliptic curve over a finite field is endowed with an addition operation “+”, under which it forms a finite abelian group.
- The group law on elliptic curves extends in a natural way to a “discrete exponentiation” on the point group of the elliptic curve.
- Based on the discrete exponentiation on an elliptic curve, one can easily derive elliptic curve analogues of the well-known public-key schemes of Diffie-Hellman and ElGamal type.

The security of such a public-key system depends on the difficulty of determining discrete logarithms in the group of points of an elliptic curve. This problem is — with current knowledge — much harder than the factorization of integers or the computation of discrete logarithms in a finite field. Indeed, since Miller and Koblitz in 1985 independently suggested the use of elliptic curves for public-key cryptographic systems, the elliptic curve discrete logarithm problem has only been shown to be solvable in certain specific, and easily recognizable, cases. There has been no substantial progress in finding an efficient method for solving the elliptic curve discrete logarithm problem on arbitrary elliptic curves. Thus, it is possible for elliptic curve based public-key systems to use much shorter parameters than the RSA system or the classical discrete logarithm based systems that make use of the multiplicative group of a finite field. This yields significantly shorter digital signatures and system parameters.

This part of ISO/IEC 15946 describes elliptic curve generation techniques useful for implementing the elliptic curve based mechanisms defined in ISO/IEC 9796-3, ISO/IEC 11770-3, ISO/IEC 14888-3, and ISO/IEC 18033-2.

It is the purpose of this part of ISO/IEC 15946 to meet the increasing interest in elliptic curve based public-key technology by describing elliptic curve generation methods to support key-exchange, key-transport and digital signatures based on an elliptic curve.

Information technology — Security techniques — Cryptographic techniques based on elliptic curves —

Part 5: Elliptic curve generation

1 Scope

ISO/IEC 15946 specifies public-key cryptographic techniques based on elliptic curves.

This part of ISO/IEC 15946 defines elliptic curve generation techniques useful for implementing the elliptic curve based mechanisms defined in ISO/IEC 9796-3, ISO/IEC 11770-3, ISO/IEC 14888-3 and ISO/IEC 18033-2.

The scope of this part of ISO/IEC 15946 is restricted to cryptographic techniques based on elliptic curves defined over finite fields of prime power order (including the special cases of prime order and characteristic two). The representation of elements of the underlying finite field (i.e. which basis is used) is outside the scope of this part of ISO/IEC 15946.

ISO/IEC 15946 does not specify the implementation of the techniques it defines. Interoperability of products complying with ISO/IEC 15946 will not be guaranteed.

2 Normative reference(s)

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/IEC 15946-1, *Information technology — Security techniques — Cryptographic techniques based on elliptic curves — Part 1: General*